Indian Association for the Cultivation of Science
Kolkata 700032, India

First Year, Second Semester
2020-2021

PHS – 1201: ELECTRICITY, MAGNETISM & OPTICS

Assignment #1 - Electrostatics

UNDERSTANDING BASIC FEATURES OF ELECTROSTATICS

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DATE: *19th April, 2021*

DUE : *25th April, 2021*

INSTRUCTIONS

- You are free to discuss the questions among yourselves if you choose to do so. However, each one should write the answers independently at the end and submit it. You should be prepared to explain the steps and arguments in your answer if called upon to do so.
- Interpret the questions as a physicist; not as a mathematician. Make reasonable assumptions when required and mention them.
- Your answers can be brief and to the point, giving just the essential algebraic steps and arguments. The marks for each of the questions are given at the right end of the question. Total marks: **60**.
- I have tried to keep the questions clear, consistent with the notation used in the class and error-free. But if you have any difficulties on these counts, feel free to email me.

1. Find the electric field at a point z above an infinite, uniform, surface charge distribution by the following two methods:
   - using Gauss's Law. **[3]**
   - not using Gauss's Law. **[6]**

   Does the expression for the electric field matches with the one derived from the potential? **[1]**

2. Find the electric field for points lying both outside and inside of a sphere with uniform volume charge distribution, without using Gauss's Law. Compare the results with the expressions derived in the class. **[10]**

3. Find the electrostatic potential of a uniformly charged spherical shell of radius R for points lying both outside and inside the shell. **[10]**

4. Imagine that extraordinary and precise measurements have revealed an error in the Coulomb's law. Such that the actual force of interaction between two point charges is found to be

$$\vec{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\left(1 + \frac{r}{\lambda}\right) e^{-\frac{r}{\lambda}}\hat{r}$$

   where λ is a new constant of nature. You are charged with the task of reformulating electrostatics to accommodate the new discovery. Assume the principle of superposition still holds.

   a) What is the electric field of a charge distribution ρ? **[5]**

   b) Does this electric field admit a scalar potential? Explain briefly how you reached your conclusion. **[5]**

   c) Find the potential of a point charge q. **[5]**

   d) For a point charge q at the origin, show that,

$$\oint_S \vec{E}\cdot d\vec{a} + \frac{1}{\lambda^2}\int_V V d\tau = \frac{q}{\epsilon_0}$$

   where S is the surface, and V the volume of any sphere centered at q. **[5]**

   e) Show that it generalizes:

$$\oint_S \vec{E}.d\vec{a} + \frac{1}{\lambda^2}\int_V V d\tau = \frac{Q_{enc}}{\epsilon_0}$$

   for any charge distribution. **[5]**

5. If the electric field in some region is given (in spherical coordinates) by the expression

$$\vec{E} = \frac{A\,\hat{r} + B\sin\theta\cos\phi\,\hat{\phi}}{r}$$

   where A and B are constants, what is the associated charge density? **[5]**