Indian Association for the Cultivation of Science
(Deemed to be University under the *de novo* category)

Master's/Integrated Master's-PhD Program/PhD Course

Mid-Semester (Sem-IV) Examination-Spring 2020

Subject: Physics of Black Holes and Gravitational Waves ***Subject Code(s): PHS 5207***
PHD 224

Full marks: 25 ***Time allotted: 2 hr***

Attempt any five questions

1. Consider a congruence of time-like geodesics characterized by the tangent vector u^a. Define the expansion, shear and rotation associated with these time-like geodesics. Hence derive the Raychaudhuri equation associated with the rate of change of the expansion scalar along the geodesics. **(5 marks)**

2. Consider a congruence of time-like geodesics, which are hypersurface orthogonal. Argue that when dominant energy condition is satisfied, i.e., $\{T_{ab} - (1/2)Tg_{ab}\}u^a u^b \geq 0$, the congruence will converge within a finite proper time bounded by $3|\theta_0|^{-1}$, where θ_0 is the initial expansion scalar, taken to be negative. **(5 marks)**

3. Consider a congruence of affinely parametrized null geodesics characterized by tangent vector ℓ^a. Show that the expansion scalar θ determines the rate of expansion of the geodesic area. **(5 marks)**

4. Write down the Schwarzschild spacetime in (t, r, θ, ϕ) system of coordinates. Define the tortoise coordinate r^* and hence show that the surface $(t - r^*) = 0$ is a null surface. Find out the metric induced on this surface and the auxiliary null vector k^a. Is the auxiliary null vector k^a affinely parametrized? **(5 marks)**

5. Draw the Penrose diagram for Schwarzschild spacetime and describe various points and lines appearing in the diagram. **(5 marks)**

6. Consider a null hypersurface $\mathcal{N}$, denoted by $\Phi(x^a) = 0$, which need not be within a family of null hypersurfaces. The normal vector to this hypersurface is in general denoted by $\ell_a = A(x^c)\nabla_a\Phi$. Show that this null vector satisfies the geodesic equation $\ell^a\nabla_a\ell^b = \kappa\ell^b$. Find out κ in terms of A and ℓ^a. Is the null vector ℓ^a affinely parameterized? **(5 marks)**

7. Given a Killing vector field ξ^a associated with a null hypersurface $\mathcal{N}$, satisfying the relation $\xi^a\nabla_a\xi^b = \kappa\xi^b$, where κ is defined as the surface gravity of the null hypersurface $\mathcal{N}$. Thus show that $\kappa^2 = -(1/2)(\nabla_a\xi_b)(\nabla^a\xi^b)$. **(5 marks)**

8. Argue that the Kerr spacetime has two Killing vectors $\xi^a_{(t)} = (\partial/\partial t)^a$ and $\xi^a_{(\phi)} = (\partial/\partial\phi)^a$. Find out the radius on which $\xi^a_{(t)}$ is null and does it represent a Killing horizon? **(5 marks)**