Lecture 8: Optical Activity and Laser

1 Optical Activity

If a plane polarized electromagnetic wave passes through certain crystalline substances, like quartz, along the optic axis, the polarization vector appears to have rotated. The angle of rotation depends on the thickness and nature of the crystal and also on the wavelength of the light employed. This phenomenon is called optical activity and the substance displaying such a phenomenon is called an optically active substance. Let us discuss a possible explanation for this phenomenon following Fresnel.

For this purpose we will use the result that any plane polarized electromagnetic wave can be equivalently considered as a superposition of two circularly polarized electromagnetic wave. To see this let us consider a light ray with the polarization vector along the x direction, such that,

$$\vec{E}_{\text{in}} = 2a\cos(kz - \omega t)\hat{x} \ . \tag{1}$$

As mentioned earlier, this plane polarized electromagnetic wave can be thought of as a combination of right circular and left circular waves. The left circularly polarized light can be expressed as

$$\vec{E}_1^{\text{in}} = \begin{cases} E_{1,x}^{\text{in}} & = a\cos(kz - \omega t) \\ E_{1,y}^{\text{in}} & = -a\sin(kz - \omega t) \ . \end{cases} \tag{2}$$

Similarly, the right circularly polarized light can be expressed as

$$\vec{E}_2^{\text{in}} = \begin{cases} E_{2,x}^{\text{in}} & = a\cos(kz - \omega t) \\ E_{2,y}^{\text{in}} & = a\sin(kz - \omega t) \ . \end{cases} \tag{3}$$

Note that, $\vec{E}_1^{\text{in}} + \vec{E}_2^{\text{in}} = \vec{E}_{\text{in}}$, which shows that any linearly polarized light can be decomposed into two circularly polarized light of opposite nature. The explanation of optical activity by Fresnel was the realization of the simple fact that these two circular components may travel through the crystal with different velocities. As a result, as they come out of the crystal, they experience a phase difference δ between them. Thus the electric fields/polarization vectors after exiting from the crystal become,

$$\vec{E}_1^{\text{out}} = \begin{cases} E_{x,1}^{\text{out}} & = a\cos(kz - \omega t) \\ E_{y,1}^{\text{out}} & = -a\sin(kz - \omega t) \ , \end{cases} \tag{4}$$

for left circular polarization and

$$\vec{E}_2^{\rm out} = \begin{cases} E_{x,2}^{\rm out} & = a\cos(kz - \omega t + \delta) \\ E_{y,2}^{\rm out} & = a\sin(kz - \omega t + \delta)\ , \end{cases} \tag{5}$$

for the right circularly polarized light. Thus the combined polarization vector along the x and the y direction, on the emergence from the crystal, becomes,

$$\begin{aligned} E_x^{\rm out} = E_{x,1}^{\rm out} + E_{x,2}^{\rm out} &= a\left[\cos(kz-\omega t) + \cos(kz - \omega t + \delta)\right] \\ &= 2a\cos\left(\frac{\delta}{2}\right)\cos\left(kz - \omega t + \frac{\delta}{2}\right)\ , \end{aligned} \tag{6}$$

$$\begin{aligned} E_y^{\rm out} = E_{y,1}^{\rm out} + E_{y,2}^{\rm out} &= a\left[\sin(kz-\omega t+\delta) - \sin(kz - \omega t)\right] \\ &= 2a\sin\left(\frac{\delta}{2}\right)\cos\left(kz - \omega t + \frac{\delta}{2}\right)\ . \end{aligned} \tag{7}$$

Hence, we have the magnitude of the polarization vector to change with time with the same frequency ω, but the direction will be time-independent, though rotated by an angle $(\delta/2)$ in an anti-clockwise direction with respect to the initial direction, i.e., x-axis, such that,

$$\frac{E_y^{\rm out}}{E_x^{\rm out}} = \tan\left(\frac{\delta}{2}\right)\ . \tag{8}$$

Thus the emergent electromagnetic wave is also plane polarized but the polarization vector is along the angle $(\delta/2)$ with the x axis. Hence different velocity for left and right circular polarization of the electromagnetic wave can indeed explain the phenomenon of optical activity.

If the velocities of the left and right circularly polarized light are $v_{\rm L}$ and $v_{\rm R}$ respectively, then the phase difference between the two circularly polarized light, after travelling through a distance ℓ, will be

$$\delta = \frac{2\pi}{\lambda} \times \left(\frac{\ell}{v_L} - \frac{\ell}{v_R}\right) \times c = \frac{2\pi\ell}{\lambda}\left(n_L - n_R\right)\ , \tag{9}$$

where, $n_{\rm L} = (c/v_{\rm L})$ and $n_{\rm R} = (c/v_{\rm R})$ are respectively the refractive index of the medium for left and right circularly polarized light respectively.

2 Laser — Light amplification by stimulated emmission of radiation

In this section we will discuss the basic operating principle for Lasers. At the most basic level, the Laser involves a two-level system which has energies E_1 and E_2 respectively. There can be three processes by which the energy can enter or exit the system.

1. **Absorption:** The atom at the ground state can absorb the incident radiation and thus transit to the excited state.

2. **Spontaneous Emission:** The atom at the excited state decays to the ground state and hence emits electromagnetic radiation.

3. **Stimulated emission:** The photon in the incident radiation forces the atom at an excited state to jump to the ground state and as a consequence emits another photon. The emitted photon will have identical direction and similar phase to the incident photon. As we will see, the stimulated emission plays a central role in the analysis.

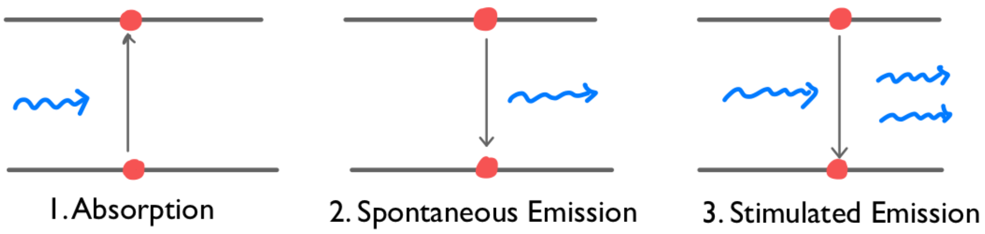

Figure 1: The three processes by which electromagnetic radiation interacts with a two-level system.

2.1 Population Inversion

Suppose we have atoms in both the energy states E_1 and E_2 and let N_1 and N_2, respectively, be the number of particles in them. If these atoms are in thermal equilibrium at the temperature T, it follows that, number of the particles in any of these states will follow the Boltzmann distribution

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{k_B T}\right) , \tag{10}$$

where, we have assumed $E_2 > E_1$. It is evident from the above expression that at finite temperature, $(N_2/N_1) \ll 1$. Only when $T \to \infty$, i.e., at very high temperature, we have $(N_2/N_1) \to 1$. Thus the probability of any transition, including spontaneous emission, depends only on the properties of the energy states E_2 and E_1 and is independent of the incident photon density. Thus in the equilibrium scenario there is no possibility to have $N_2 > N_1$. While for our purpose, we need $N_2 > N_1$ and this scenario with $N_2 > N_1$ is referred to as the population inversion and as evident that this cannot be achieved in equilibrium. In practice, as we will discuss later, population inversion is achieved by pumping, which is a non-equilibrium process and hence the Boltzmann's law is not applicable leading to $N_2 > N_1$.

2.2 Einstein's A and B coefficients

The physics of Laser can be understood using the A and B coefficients of Einstein, introduced much before to understand the emission and absorption from discrete energy levels of an atom.

Let us start with the phenomenon of spontaneous emission, introduced earlier, in which case the probability for at atom in the energy state E_2 to decay to E_1 is $A_{21} = (1/\tau_{21})$, where τ_{21} is the average lifetime of an atom to exist in the upper energy state. If N_2 is the total number of atoms per unit volume in the excited state, then it follows that the fraction of them making transitions per unit volume per unit time to the lower energy state, is $N_2 A_{21}$. Then we also have the process of absorption, in which the incident photon gets absorbed and an atom in the lower energy state make a transition to the excited state. Let B_{12} be the probability of this process. Total number of such transitions will depend on N_1, the number of particles in the lower energy state, as well as on the number of photons within the frequency range ν to $\nu + d\nu$. Thus total such transitions per unit time per unit volume becomes $N_1 B_{12} u_\nu$, where u_ν measures the energy density of photons within a frequency range ν and $\nu + d\nu$. Finally for stimulated emission we have B_{21} as the probability, for which the total number of transitions per unit time per unit volume becomes $N_2 B_{21} u_\nu$.

For equilibrium, the total number of the transitions from energy level E_2 to energy level E_1 should be equal to the number of transitions from E_1 to E_2. Thus we obtain,

$$
\begin{aligned}
& N_1 B_{12} u_\nu = N_2 A_{21} + N_2 B_{21} u_\nu \\
\Longrightarrow\ & u_\nu (N_1 B_{12} - N_2 B_{21}) = N_2 A_{21} \\
\Longrightarrow\ & u_\nu = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}} \\
\Longrightarrow\ & u_\nu = \frac{\dfrac{A_{21}}{B_{21}}}{\dfrac{N_1}{N_2}\dfrac{B_{12}}{B_{21}} - 1} \,.
\end{aligned}
\tag{11}
$$

In equilibrium, following the Boltzmann distribution at temperature T, we have, $(N_1/N_2) = \exp[-(E_1 - E_2)/k_B T] \equiv \exp(h\nu/k_B T)$, where $h\nu \equiv E_2 - E_1$. Therefore, the above result becomes

$$
u_\nu = \frac{\dfrac{A_{21}}{B_{21}}}{\dfrac{B_{12}}{B_{21}} e^{h\nu/k_B T} - 1}
\tag{12}
$$

If we now use the fact that a photon gas in equilibrium at a temperature T, is distributed in a blackbody spectrum, it follows that,

$$
u_\nu = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/k_B T} - 1}
\tag{13}
$$

From these two expressions for u_ν, we can conclude that the coefficients A and B defined above satisfies the conditions,

$$
\frac{B_{12}}{B_{21}} = 1 \,; \qquad \frac{A_{21}}{B_{21}} = \frac{8\pi h \nu^3}{c^3} \,.
\tag{14}
$$

These two relations are known as the Einstein's relations and these coefficients are referred to as the Einstein's A and B coefficients. This signifies that the probability of absorption is same as the probability of the stimulated emission as $B_{12} = B_{21}$. Finally, the ratio of the spontaneous emission to the stimulated emission, for the thermal equilibrium is given by,

$$R = \frac{A_{21}N_2}{B_{21}N_2u_\nu} = \frac{A_{21}}{B_{21}u_\nu} = e^{h\nu/k_BT} - 1 \; . \tag{15}$$

For the visible light, it follows that $(h\nu) \gg k_BT$ and hence $R \gg 1$. While for the microwave radiation one has $h\nu \ll k_BT$ and hence $R \ll 1$. So, for the incident microwave radiation most of the emerging radiation is stimulated in nature. This is why MASER or microwave amplification has been developed much before LASER.

2.3 Basic parts of a Laser

There is a pumping source, which achieves population inversion by raising atoms from ground state to the higher excited state by supplying energy from an external source. The radiation produced after achieving the population inversion will be mostly stimulated, as one photon can lead to an avalanche of stimulated photons. Number of such stimulated photons are increased by reflecting the photons back and forth in the optical cavity. The source (pump) is switched on in a periodic manner to achieve population inversion once again. Then the same process continues. We describe below a few methods to achieve population inversion.

1. **Optical Pumping:** The atoms are exposed to strong sources of light, such as a flash lamp. By selective absorption of the radiation the atoms are excited from the lower to the higher excited state. This is called optical pumping.

2. **Collision of the first kind:** This is through gas discharge. Electrons are accelerated to a high energy by employing a strong electric field. They will collide with the gas atoms and hence transfer the kinetic energy, lifting the atoms to a higher energy state. The process will look like

$$A + e_{\text{fast}} \to A^* + e_{\text{slow}} \tag{16}$$

where A^* is the atom in an excited state.

3. **Collisions at the second kind:** In this case one use discharge tube but for a gas mixture $(A + B)$. Suppose A has a metastable state A^* and A goes to A^* by collision with the electron arising out of the gas discharge. Then A^* will interact with B and $B \to B^*$, while A goes to the ground state. In this way, the gas component B can be excited to the higher energy state B.

4. Finally, one can also employ certain exothermal reactions acting as the pumping mechanism.

2.4 Threshold condition for Laser

Suppose a Laser light is passing through a medium where population inversion has been achieved. Then energy loss due to stimulated absorption is given by,

$$B_{12}N_1 \times h\nu \times u_\nu \times Sdx \tag{17}$$

where, Sdx is an infinitesimal volume element of the material with surface area S and width dx. On the other hand, the energy gain due to stimulated emission is

$$B_{21}N_2\, h\nu\, u_\nu Sdx \tag{18}$$

Hence the rate of increase of energy in this volume element Sdx is given by,

$$\Delta E_{\text{in}} = B_{12}\left(N_2 - N_1\right) h\nu u_\nu Sdx \tag{19}$$

On the other hand, if I_ν is the intensity of the Laser light leaving the system, then the net energy leaving the system is given by,

$$\Delta E_{\text{out}} = \frac{dI_\nu}{dx} Sdx \tag{20}$$

Since total energy is conserved, it follows that $\Delta E_{\text{in}} = \Delta E_{\text{out}}$ and hence

$$\begin{aligned} \frac{dI_\nu}{dx} &= (N_2 - N_1)B_{12}\, h\nu\, u_\nu \\ \implies \frac{dI_\nu}{dt} &= \frac{dI_\nu}{dx}\frac{dx}{dt} = (N_2 - N_1)B_{12}\, h\nu\, u_\nu v_{\text{eff}} \end{aligned} \tag{21}$$

where v_{eff} is the effective velocity of the photon passing through the medium[1]. As evident, in order to have gain, i.e., for $(dI_\nu/dt) > 0$, it is necessary to have $N_2 > N_1$. Thus population inversion is absolutely essential. Of course, in realistic scenarios, there will be losses due to scattering of the photon. If t_{photon} is the mean time between two successive collision, the loss of the intensity is given by, $(\nu u_\nu c/t_{\text{photon}})$. For effective operation of the Laser, the gain in the intensity must dominate over the loss and hence, we have

$$\begin{aligned} \left(N_2 - N_1\right) B_{12} h\nu u_\nu v_{\text{eff}} &> \frac{\nu u_\nu c}{t_{\text{photon}}} \\ \implies N_2 - N_1 &> \frac{c}{B_{12}h\left(v_{\text{eff}} t_{\text{photon}}\right)} \end{aligned} \tag{22}$$

Substituting for B_{12} from the Einstein's relations we obtain,

$$N_2 - N_1 > \frac{8\pi\nu^3}{\left(v_{\text{eff}} t_{\text{photon}}\right) c^2 A_{21}} = \frac{8\pi\nu^3 \tau_{21}}{\left(v_{\text{eff}} t_{\text{photon}}\right) c^2} \tag{23}$$

where τ_{21} is the decay time scale of an atom from the excited state to a lower energy state.

[1] Velocity of a photon is always c