Lecture 3: Observational Avenues in Interference

1 Newton's Ring

Newton's rings are a particular example of interference fringes formed by thin films. By placing plano-convex lens on a plane glass plate, a film of gradually increasing thickness from the point of contact O can be formed as shown in Figure 1. If it is illuminated by a monochromatic light, interference fringes in the form of concentric circular rings can be observed. These rings are known as "Newton's rings". These fringes are the loci of points of equal film thickness.

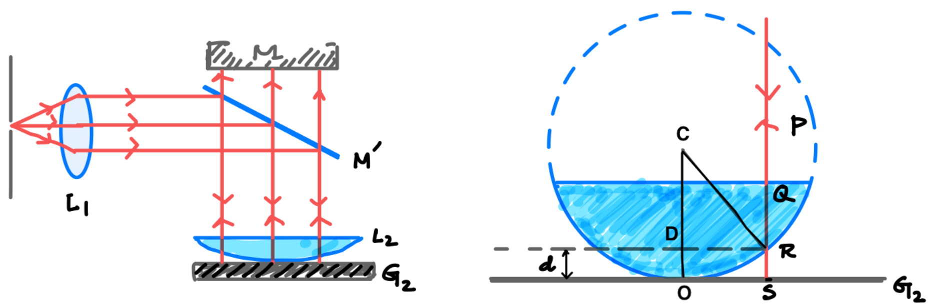

Figure 1: Set up for the Newton's ring experiment.

Conditions for interference fringe pattern: As shown in Figure 1, the incident light ray falls normally along PQR and partially reflects back from R. The transmitted part gets reflected back from S from the upper surface of the glass plate G_2 and superposed with the ray that is reflected from R. As a result the path difference between the two rays becomes $2d$, where d is the width of variable air film at that point. But the transmitted ray is reflected back by a denser medium. Hence the optical path difference should be

$$\Delta = 2d \pm \frac{\lambda}{2} \; . \tag{1}$$

Hence, the condition for the formation of bright fringes is given by

$$2d \pm \frac{\lambda}{2} = 2n \times \frac{\lambda}{2}$$

$$\implies 2d = (2n \pm 1)\,\frac{\lambda}{2}\ , \tag{2}$$

where, n is a positive integer. Similarly, the condition for the formation of dark fringes becomes

$$2d = 2n\left(\frac{\lambda}{2}\right)\ , \tag{3}$$

where, n is again a positive integer. Therefore, if the width of the air film is even multiple of $\lambda/2$, then it will form dark fringes, while if the width of the film is an odd multiple of $\lambda/2$, then it will form bright fringes.

Radius of concentric fringes: A fringe of order m will be along the loci of the points of equal thickness d and hence the fringes will be circular in nature. From the geometry of Figure 1, we have $CR = R$, where R is the radius of curvature of the lens and C is the centre of the circle, depicted on the right side of Figure 1. Also we have $CD = R - d$. Let the radius of the m^{th} order bright/dark fringe be r_{m}. So, we have $DR = r_{\mathrm{m}}$. Now from the triangle ΔCDR, we have

$$\begin{aligned} R^2 &= (R-d)^2 + r_{\mathrm{m}}^2 \\ \implies R^2 - (R-d)^2 &= r_{\mathrm{m}}^2 \\ \implies r_{\mathrm{m}}^2 &= d(2R-d) \approx 2Rd \qquad [\text{as } R \gg d]\ . \end{aligned} \tag{4}$$

Hence, the radius of the $\mathrm{m^{th}}$ order bright and dark fringe is given by

$$r_{\mathrm{m}}^{\mathrm{bright}} = \sqrt{\frac{(2m \pm 1)\lambda R}{2}}\ ; \tag{5}$$

$$r_{\mathrm{m}}^{\mathrm{dark}} = \sqrt{\frac{2m\lambda R}{2}} = \sqrt{m\lambda R}\ . \tag{6}$$

Fringe width of Newton's ring: Usually, we express the fringe width of Newton's ring in terms of the diameter of the ring. Let the diameter of the $\mathrm{m^{th}}$ order fringe be D_{m}. Then the fringe width is given by

$$\beta_{\mathrm{m}} = r_{\mathrm{m+1}} - r_{\mathrm{m}} = \frac{r_{\mathrm{m+1}}^2 - r_{\mathrm{m}}^2}{r_{\mathrm{m+1}} + r_{\mathrm{m}}} \approx \frac{\dfrac{\lambda R}{2} \times 2}{2r_{\mathrm{m}}} = \frac{\lambda R}{D_{\mathrm{m}}}\ . \tag{7}$$

Nature of the central fringe: For central fringe, there is no air gap i.e., $d = 0$. Hence the order number of the fringe is $n = 0$. This reflects the fact that the central fringe will be dark in nature.

Replacing with a medium of refractive index μ: If we replace the air medium present in the gap between plano-convex lens and the glass, by some other material with refractive index

μ (less than the refractive index of the glass) then the optical path difference will become $2\mu d$. Thus the radius of the m^{th} order fringe will be

$$r_{\mathrm{m}}^{\mathrm{bright}} = \sqrt{\frac{(2m+1)\lambda R}{2\mu}} \; ; \tag{8}$$

$$r_{\mathrm{m}}^{\mathrm{dark}} = \sqrt{\frac{m\lambda R}{\mu}} \; . \tag{9}$$

And this change in the radius of the fringes will also cause a change in the fringe width, which is given by,

$$\beta_{\mathrm{m}} = \frac{\lambda R}{\mu D_{\mathrm{m}}} \; . \tag{10}$$

Applications: As discussed earlier, changing the material of the gap between plano-convex lens and the glass, changes the fringe width. Hence by measuring the fringe width for different materials we can determine their refractive indices. Wavelength of a monochromatic light ray can also be determined by performing the Newton's ring experiment.

2 Michelson Interferometer

The Michelson interferometer is an amplitude division class of interferometer i.e., the interference pattern is observed due to the division of the amplitude of the incident light, which are made to interfere later. This interferometer is an arrangement of mirrors and beam splitters, see Figure 2. There are two mirrors M_1 and M_2 in this set up, out of which the mirror M_2 can move. The light ray is reflected from A travels through the glass thrice before reaching the screen. So, another glass slab C is often introduced in order to compensate for the additional path length along M_2.

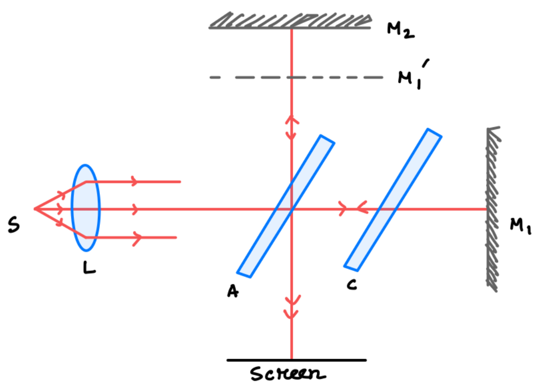

Figure 2: Michelson interferometer set up.

Here the path difference is given by $\Delta = 2d\cos\theta$ (see Figure 3). If the bean splitter is an uncoated slab of glass, then the ray to mirror M_2 gets reflected from denser to rarer medium, while the ray from mirror M_1 is reflected from rarer to denser medium. Thus there will a phase difference of π and the condition of destructive interference will be $2d\cos\theta$ being some integral multiple of λ. On the other hand, if the bean splitter has some coating on its one side, then both the reflections are from rarer to denser medium and hence no relative phase difference exists. Thus for constructive interference we have the following condition

$$2d\cos\theta = m\lambda \ . \tag{11}$$

where m is an integer. For a fixed value of the angle, the fringes form a circle and hence

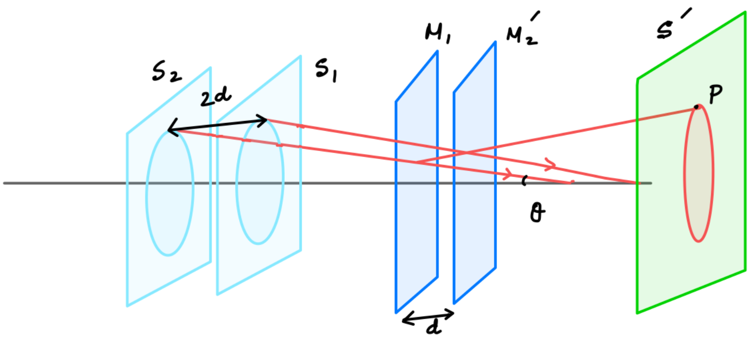

Figure 3: Diagrammatic representation of the formation of circular fringes.

concentric fringes are concentric circles in this case. The two sources S_1 and S_2, which are co-linear, combine to give total intensity at P. Thus for constructive interference, we obtain (recall the computation in the first lecture),

$$\begin{aligned}
&\sqrt{x^2+y^2+(D+d)^2} - \sqrt{x^2+y^2+(D-d)^2} = 2m\,(\lambda/2) = m\lambda \\
\Longrightarrow\ & x^2+y^2+(D+d)^2 = m^2\lambda^2 + x^2+y^2+(D-d)^2 + 2m\lambda\sqrt{x^2+y^2+(D-d)^2} \\
\Longrightarrow\ & 2m\lambda\sqrt{x^2+y^2+(D-d)^2} = 4Dd - m^2\lambda^2 \\
\Longrightarrow\ & x^2+y^2+D^2+d^2-2Dd = \frac{(4Dd-m^2\lambda^2)^2}{4m^2\lambda^2} = \frac{4D^2d^2}{m^2\lambda^2} - 2Dd + \frac{m^2\lambda^2}{4} \\
\Longrightarrow\ & x^2+y^2 = D^2\left(\frac{4d^2}{m^2\lambda^2}-1\right) + \left(\frac{m^2\lambda^2}{4} - d^2\right) \\
\Longrightarrow\ & x^2+y^2 = \left(D^2 - \frac{m^2\lambda^2}{4}\right)\left(\frac{4d^2}{m^2\lambda^2}-1\right) \\
\Longrightarrow\ & x^2+y^2 \approx D^2\left(\frac{4d^2}{m^2\lambda^2}-1\right) \qquad [\text{as}\ \ D^2 \gg m^2\lambda^2/4] \\
\Longrightarrow\ & r_{\mathrm{m}}^{\mathrm{bright}} = D\sqrt{\left(\frac{4d^2}{m^2\lambda^2}-1\right)} \ .
\end{aligned} \tag{12}$$

For dark fringes to be formed, the path difference should be $(m+1/2)\lambda$. Thus the radius of the dark circular fringes is given by

$$r_{\mathrm{m}}^{\mathrm{dark}} = D\sqrt{\left[\frac{4d^2}{\left(m+\frac{1}{2}\right)^2\lambda^2} - 1\right]} \; . \tag{13}$$

3 Applications of Michelson Interferometer

1. **Finding wavelength of a monochromatic light:** For Michelson interferometer, shifting the movable mirror causes a shift in the fringe position. If we move the mirror by a distance d, and as a result m number of fringes passes the cross-wire. Then we have,

$$2d = m\lambda \quad [\text{here}, \theta \approx 0] \; . \tag{14}$$

As we can measure m and the amount of distance the mirror got shifted, we can easily calculate the wavelength of the light ray.

2. **Determination of small difference in wavelength:** In case of a not-so-pure monochromatic light source, Michelson's interferometer can be used to determine the small difference in the wavelength. If the maxima of one wavelength coincides with the minima of the other, then the fringes will be indistinct (known as disonance). While, if the maxima of one falls on the maxima of the other, then the fringes become distinct (known as consonance).

Let us shift the mirror by a distance d, such that they move from one consonance to another through disonance. Let the position of the mirror before and after the shift, be x_1 and x_2 respectively. Thus $d = x_2 - x_1$. Further, suppose before the shift of the mirror, the $\mathrm{m}_1^{\mathrm{th}}$ order fringe of one wavelength λ_1 matches with $\mathrm{m}_2^{\mathrm{th}}$ order fringe of the other wavelength λ_2. Similarly after the shift, the $\mathrm{n}_1^{\mathrm{th}}$ order fringe of one wavelength matches with the $\mathrm{n}_2^{\mathrm{th}}$ order fringe of the other wavelength. Then we have,

$$2x_1 = m_1\lambda_1 = m_2\lambda_2 \; , \tag{15}$$
$$2x_2 = n_1\lambda_1 = n_2\lambda_2 \; . \tag{16}$$

This yields,

$$2d \equiv 2(x_2 - x_1) = \Delta m\lambda_1 = \Delta n\lambda_2 \; , \tag{17}$$

where $\Delta m = n_1 - m_1$ and $\Delta n = n_2 - m_2$. As discussed before, the transition from one consnance to another requires, $\Delta n = \Delta m + 1$ and this leads to,

$$\begin{aligned} \frac{2d}{\lambda_2} - \frac{2d}{\lambda_1} &= 1 \\ \implies \frac{2d}{\lambda_1\lambda_2}\Delta\lambda &= 1 \\ \implies \Delta\lambda &= \frac{\lambda^2}{2d} \; , \end{aligned} \tag{18}$$

where, we have assumed $\lambda_1 \approx \lambda_2 = \lambda$.

3. **Refractive index and thickness of a film:** To determine the refractive index or the thickness of a thin film, the film is placed in the path of the interfering rays. As a result an extra $(\mu-1)t$ optical path difference will be introduced, where μ is the refractive index of the film and t is its thickness. This in turn causes a shift of the central fringe, which is again made coincident with the cross wire by moving the mirror M_2 by a distance d. In that case (assuming $\theta \sim 0$), we have,

$$\begin{aligned} 2d &= 2(\mu-1)t \\ \implies d &= (\mu-1)t \ . \end{aligned} \tag{19}$$

So if we have the information of either the refractive index or the thickness of the film, then the other one can be easily determined.

4 Fabry-Perot Interferometer

The Fabry-Perot interferometer is a high resolving power interferometer. It creates fringes of equal inclination that are produced by the transmitted light rays after multiple reflection in an air gap between two parallel reflecting glass plates. This interferometer consists of two glass slabs $ABCD$ and $EFGH$. The part of the mirrors AB and EF are silver-coated and parallel to each other (see Figure 4). However, parts of the individual glass slabs are not parallel, for example, AB and CD are not parallel. Similarly EF and HG are also not parallel. This is made so to keep away the stray light from multiple reflections within the slab itself. There is an air film between AB and EF of width d. So, after multiple reflection when the transmitted rays get superposed, they develop a path difference of $2d\cos\theta$, where θ is the angle made by the ray at the air film (see Figure 4). This is like the amplitude division class of interference. Suppose

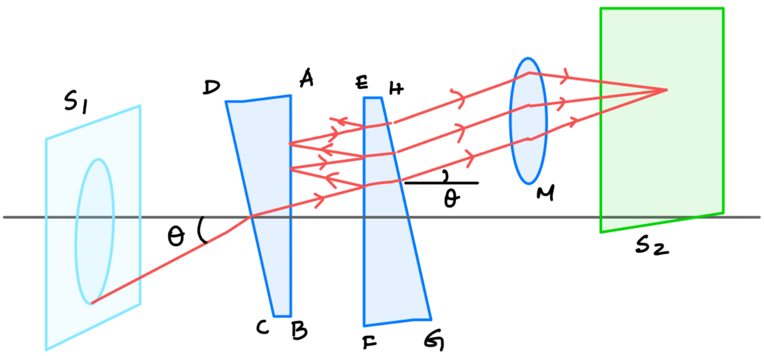

Figure 4: Set up of Fabry-Perot interferometer.

the amplitude of the incident wave with angle of incidence θ be A. Let r and r' be the reflection coefficients for the waves reflected from the outer and inner surfaces respectively. Similarly, let t and t' be the transmission coefficients for the waves transmitted inside and outside the film respectively. Then from Stoke's theorem, we have

$$r = -r' \ , \tag{20}$$

$$r^2 = 1 - tt' \ . \tag{21}$$

Therefore, the amplitudes of the first, second, third, ... transmitted rays will be Att', $Att'r'^2e^{i\delta}$, $Att'r'^4e^{2i\delta}$, $\cdots$, respectively where δ is the phase difference between two successive rays. The phase difference δ is given by

$$\delta = \frac{2\pi}{\lambda} \times 2d\cos\theta \ . \tag{22}$$

If the angle of incidence is small then there will be large number of transmitted waves and the resultant amplitude of the transmitted rays will be

$$\begin{aligned}
A_{\mathrm{T}} &= Att'(1 + r'^2e^{i\delta} + r'^4e^{2i\delta} + \cdots) \\
&= A \times (1 - r^2) \times \frac{1}{1 - r^2e^{i\delta}} \qquad [\text{As, } \ tt' = 1 - r^2, r = -r'] \\
&= A\left(\frac{1 - r^2}{1 - r^2e^{i\delta}}\right) \ .
\end{aligned} \tag{23}$$

Hence, the resultant intensity of the transmitted wave is,

$$\begin{aligned}
I_{\mathrm{T}} = |A_{\mathrm{T}}|^2 = |A|^2\frac{(1 - r^2)^2}{(1 + r^4) - 2r^2\cos\delta} \\
= I\left\{\frac{(1 - r^2)^2}{(1 - r^2)^2 + 4r^2\sin^2\left(\frac{\delta}{2}\right)}\right\} \\
= I\left[1 + \frac{4r^2}{(1 - r^2)^2}\sin^2\left(\frac{\delta}{2}\right)\right]^{-1} \ ,
\end{aligned} \tag{24}$$

where, $I = |A|^2$ is the intensity of the incident light. For notational convenience, let us define $F \equiv \{4r^2/(1 - r^2)^2\}$, which yields,

$$\frac{I_{\mathrm{T}}}{I} = \left[1 + F\sin^2\left(\frac{\delta}{2}\right)\right]^{-1} \tag{25}$$

It is evident that the resultant intensity varies with the phase difference. When phase difference is integral multiple of 2π, we have maximum intensity, given by $(I_{\mathrm{T}}/I)_{\max} = 1$. As a special case, let us define the phase difference to be $\delta_{1/2}$, when we have $I_{\mathrm{T}} = (I/2)$. Thus $\delta_{1/2}$ reads,

$$\begin{aligned}
\frac{1}{2} &= \frac{1}{1 + F\sin^2\left(\frac{\delta_{1/2}}{2}\right)} \\
\Longrightarrow F\sin^2\left(\frac{\delta_{1/2}}{2}\right) &= 1 \\
\Longrightarrow \delta_{1/2} &= 2\sin^{-1}\left(\frac{1}{\sqrt{F}}\right) \ .
\end{aligned} \tag{26}$$

On the other hand, if the phase difference is an odd integer multiple of π, then we have the minimum intensity of the transmitted light, given by

$$\left(\frac{I_{\mathrm{T}}}{I}\right)_{\min} = \frac{1}{(1 + F)} = \left(\frac{1 - r^2}{1 + r^2}\right)^2 \ . \tag{27}$$

In this context, we can define the visibility (V) of the fringes as,

$$\begin{aligned} V &= \frac{(I_{\mathrm{T}}/I)_{\max} - (I_{\mathrm{T}}/I)_{\min}}{(I_{\mathrm{T}}/I)_{\max} + (I_{\mathrm{T}}/I)_{\min}} \\ &= \frac{1 - \dfrac{1}{1+F}}{1 + \dfrac{1}{1+F}} = \frac{F}{2+F} \\ &= \frac{4r^2}{2(1-r^2)^2 + 4r^2} = \frac{2r^2}{1+r^4} \ . \end{aligned} \tag{28}$$

5 Applications of the Fabry-Perot Interferometer

1. **Comparison of wavelength:** Suppose the light source used in the interferometer, contains two closely spaced wavelengths λ_1 and λ_2 respectively. Then the interferometer is adjusted, such that the maxima of one wavelength coincides with the maxima of the other wavelength. Suppose, when the width of the air gap is d_1 and $\mathrm{m}_1^{\mathrm{th}}$ order fringe of wavelength λ_1 coincides with the $\mathrm{m}_2^{\mathrm{th}}$ order fringe of the wavelength λ_2. This yields,

$$2d_1 = m_1\lambda_1 = m_2\lambda_2 \ . \tag{29}$$

Suppose, the width of the gap is increased to d_2 from d_1 such that the coincidence of the maxima is achieved again. So, now let $(m_1+p)^{\mathrm{th}}$ order fringe of wavelength λ_1 coincides with the $(m_2+p+1)^{\mathrm{th}}$ order fringe of the wavelength λ_2. Hence

$$2d_2 = (m_1+p)\lambda_1 = (m_2+p+1)\lambda_2 \ . \tag{30}$$

Comparing the above two equations, we obtain,

$$\begin{aligned} &2(d_2-d_1) = p\lambda_1 = (p+1)\lambda_2 \\ \implies &\frac{2(d_2-d_1)}{\lambda_2} - \frac{2(d_2-d_1)}{\lambda_1} = 1 \\ \implies &\Delta\lambda = \frac{\lambda_1\lambda_2}{2(d_2-d_1)} \ . \end{aligned} \tag{31}$$

This result can be used to calculate the small separation of two closely spaced wavelength.

2. **Wavelength measurement:** To calculate the wavelength of the light source producing the interference patter, the interferometer is adjusted in order to have a bright fringe at the centre. Let the width of the air gap be d_1 at this moment. Then suppose the air gap is changed to d_2 so that the bright fringe reappear at the centre. Let this change in width of the air gap causes N numbers of fringe shift. Then

$$2(d_1-d_2) = N\lambda \ . \tag{32}$$

Here $\cos\theta = 1$, as we are using the central fringe . Therefore having the knowledge of (d_1-d_2) and N, we can determine the wavelength λ of the light being emitted from the source.

3. **Study of hyperfine structure:** As Fabry-Perot interferometer has high resolving power, so it can be used to determine the hyperfine structure of spectral lines. Suppose λ_1 is the main line and $\lambda_1 - \Delta\lambda$ is the satellite line. Then,

$$\begin{aligned} &2d\cos\theta_1 = m\lambda_1 \ , \\ &2d\cos\theta_2 = (m-1)\lambda_1 \quad \text{[for next bright fringe]} \ . \end{aligned}$$

But, suppose $(m-1)^{\text{th}}$ order fringe of wavelength λ_1 coincides with m^{th} order fringe of wavelength $\lambda_1 - \Delta\lambda$. Then,

$$2d\cos\theta_2 = m(\lambda_1 - \Delta\lambda) \ . \tag{33}$$

Hence, we have

$$\begin{aligned} &\lambda_1 = m\Delta\lambda \ , \\ \Longrightarrow\ &\Delta\lambda = \frac{\lambda_1}{m} = \frac{\lambda_1^2}{2d\cos\theta_1} \approx \frac{\lambda_1^2}{2d} \ . \end{aligned} \tag{34}$$

Thus, if we have the information about d and λ_1 then the wavelength difference $\Delta\lambda$ can be easily determined.