Indian Association for the Cultivation of Science
(a deemed to be university)

Integrated M.Sc.-Ph.D. Program in Physical Sciences (IntPPS)/PhD
(In Collaboration with Jadavpur University)

Mid-Semester Examination-2019 (Spring Semester)

Paper: General Relativity and Cosmology | **Paper Code: PH-516/PHD 218**
Full Marks: 20 | **Time Allotted: 2hr**

Instructions

(a) Attempt any **four** questions.

(b) For most of the questions the algebra will be self-explanatory; when some interpretation/description is needed, you can keep it brief but clear. There is no need to provide extensive description.

(c) The marks for each question is given against the question. If you find that you cannot answer part (a), say, of a question but can answer part (b) *assuming the result in part (a)*, you may do so and you will get credit for part (b).

(e) You may find the following relations useful:

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(-\partial_\nu g_{\alpha\beta} + \partial_\beta g_{\nu\alpha} + \partial_\alpha g_{\beta\nu} \right)$$

$$\nabla_\alpha V^\beta = \partial_\alpha V^\beta + \Gamma^{\beta}_{\alpha\mu} V^\mu; \qquad \nabla_\alpha V_\beta = \partial_\alpha V_\beta - \Gamma^{\rho}_{\alpha\beta} V_\rho$$

$$\pounds_u v^a = u^c \partial_c v^a - v^c \partial_c u^a; \qquad \pounds_u w_a = u^c \partial_c w_a + w_c \partial_a u^c$$

1. An astronomical source of light is moving along a direction with velocity v making an angle θ to our line of sight. Hence show that apparent transverse speed of the source is given by,

$$v_{\text{apparent}} = \frac{v \sin\theta}{1 - (v/c)\cos\theta}$$

   Can this exceed c? Does that constitute a violation of relativity principle? **(5 marks)**

2. Suppose we make a coordinate transformation from Cartesian coordinates (x, y, z) to cylindrical coordinate system (r, ϕ, z), such that $x = r\cos\phi$ and $y = r\sin\phi$. Find out the metric in the cylindrical coordinate system by figuring out how the line element transforms to cylindrical coordinate system. Hence find out the components of Christoffel connection in cylindrical coordinate system. **(5 marks)**

3. Derive the trajectory of a particle moving with a constant acceleration a along the x-direction. Illustrate it in a spacetime diagram. **(5 marks)**

4. Suppose Alice is living throughout her life on the top floor of a building of height 100 meter. Her friend Bob, on the other hand, is living at the ground floor of the same building. Hence one second in Alice's watch, will be equivalent to what time interval of Bob's watch? If Alice lives for 100 years, what will be the age of Bob at that instant? **(5 marks)**

5. Consider a Lagrangian $L = L(q, \dot{q}, \ddot{q})$. Vary the action and determine the field equation. Find out the conjugate momentum and hence the Hamiltonian. Can you argue why the dynamics arising out of this Lagrangian is unstable. **(5 marks)**

6. Show that $\Gamma^{\alpha}_{\alpha\mu} = \partial_\mu \ln\sqrt{-g}$. Hence or otherwise show that for a symmetric tensor $S^{\mu\nu}$ the following result holds,

$$\nabla_\nu S^\nu_\mu = \frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}S^\nu_\mu\right) - \frac{1}{2}S^{\alpha\beta}\partial_\mu g_{\alpha\beta}$$

   What happens if $S^{\mu\nu}$ is replaced by an antisymmetric tensor $A^{\mu\nu}$ in the above expression? Can you mention a physical instance where the above result becomes important? **(5 marks)**

7. Suppose ξ^μ is a vector generating symmetry of the spacetime. Show this vector must satisfy the condition $\nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu = 0$. Can you solve this equation in flat spacetime and comment on its connection with Lorentz transformation? **(5 marks)**

8. Using the weak field approximation, show that the particles at rest in a gravitational field accelerates. However, the freely falling particles move in a geodesic. **(5 marks)**