Indian Association for the Cultivation of Science
(a deemed to be university)

Integrated M.Sc.-Ph.D. Program in Physical Sciences (IntPPS)
(In Collaboration with Jadavpur University)

Semester Examination-02 (Sem-IV, 2019)

Paper: General Relativity and Cosmology **Paper Code: PH-516**
Full Marks: 20 **Time Allotted: 1hr and 30mins**

Instructions

(a) Attempt any **four** questions.

(b) For most of the questions the algebra will be self-explanatory; when some interpretation/description is needed, you can keep it brief but clear. There is no need to provide extensive description.

(c) The marks for each question is given against the question. If you find that you cannot answer part (a), say, of a question but can answer part (b) *assuming the result in part (a)*, you may do so and you will get credit for part (b).

(e) You may find the following relations useful:

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(-\partial_\nu g_{\alpha\beta} + \partial_\beta g_{\nu\alpha} + \partial_\alpha g_{\beta\nu} \right)$$

$$\nabla_\alpha V^\beta = \partial_\alpha V^\beta + \Gamma^{\beta}_{\alpha\mu} V^\mu; \qquad \nabla_\alpha V_\beta = \partial_\alpha V_\beta - \Gamma^{\rho}_{\alpha\beta} V_\rho$$

1. Derive the trajectory of a charged particle with charge q, moving in a constant electric field along x-direction, whose strength is given by E. **(5 marks)**

2. Using the coordinate transformation to a rotating frame of reference, determine the associated line element. Can you find out the potential associated with rotating frame of reference. **(5 marks)**

3. Suppose Alice is living throughout her life on the top floor of a building of height 100 meter. Her friend Bob, on the other hand, is living at the ground floor of the same building. Hence one second in Alice's watch, will be equivalent to what time interval of Bob's watch? If Alice lives for 100 years, what will be the age of Bob at that instant? **(5 marks)**

4. Show that, using coordinate transformation, spacetime metric $g_{\mu\nu}$ at a point can always be reduced to $\eta_{\mu\nu}$ and $\partial_\alpha g_{\mu\nu}$ can be set to zero. But all second derivatives of the metric cannot be set to zero and the number of remaining components will be 20 in four dimensions. **(5 marks)**

5. Starting from Poisson's equation in electrostatics can you argue that, electrodynamics has to be a vector field theory? In a similar manner, show that gravity must be described by a second rank tensor field. **(5 marks)**

6. Consider the following Lagrangian density for a scalar field theory, $L = -(1/2)\partial_\mu\phi\partial^\mu\phi - V(\phi)$. Vary this Lagrangian and obtain the field equation. What will be the momenta conjugate to ϕ? For a massive particle $V(\phi) = (1/2)m^2\phi^2$, while for Higgs field $V(\phi) = -(1/2)m^2\phi^2 + \lambda\phi^4$. Determine the associated field equations. **(5 marks)**

7. Consider a Lagrangian $L = L(q, \dot{q}, \ddot{q})$. Vary the action and determine the field equation. Find out the conjugate momentum and hence the Hamiltonian. Can you argue why the dynamics arising out of this Lagrangian is unstable. **(5 marks)**

8. Generalize the Lagrangian of a relativistic point particle to $mc^2 + \lambda\phi(x)$, where $\phi(x)$ is a scalar. Derive the equation of motion of the particle and hence show that you recover the Newton's law $\ddot{\boldsymbol{x}} \propto \boldsymbol{\nabla}\phi$ in the non-relativistic limit. **(5 marks)**

9. Derive geodesic equation with affine parameter τ, starting from the action,

$$\int d\tau \left\{ -g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} \right\}$$

(5 marks)

10. Using the weak field approximation, show that the particles at rest in a gravitational field accelerates. However, the freely falling particles move in a geodesic. **(5 marks)**