Indian Association for the Cultivation of Science
(Deemed to be University under the *de novo* category)

Master's/Integrated Master's-PhD Program/PhD Course

Final-Semester (Sem-IV) Examination-Spring 2020

Subject: Physics of Black Holes and Gravitational Waves ***Subject Code(s): PHS 5207***
PHD 224

Full marks: 50 ***Time allotted: 3 hr***

Attempt any five questions

1. Consider a congruence of null geodesics characterized by the tangent vector ℓ^a. Hence derive the Raychaudhuri equation associated with the rate of change of the expansion scalar along the null geodesics. If these null geodesics are hypersurface orthogonal, argue that when null energy condition is satisfied, i.e., $T_{ab}\ell^a\ell^b \geq 0$, the congruence will converge within a finite proper time bounded by $2|\theta_0|^{-1}$, where θ_0 is the initial expansion scalar, taken to be negative. **(10 marks)**

2. Show that any static and spherically symmetric metric can be expressed as a Rindler metric in the near horizon regime. Hence determine the surface gravity and hence the temperature associated with the horizons of Schwarzschild and Reissner-Nordström spacetime. In Reissner-Nordström spacetime, what happens in the $Q \to M$ limit? Explain. **(10 marks)**

3. Draw the Penrose diagram of the Reissner-Nordström-de Sitter spacetime, whose metric elements in the standard (t, r, θ, ϕ) coordinate system is: $g_{ab} = \text{diag.}(-f(r), (1/f(r)), r^2, r^2 \sin^2\theta)$, where $f(r) = 1 - (2M/r) + (Q^2/r^2) - (\Lambda/3)r^2$. Here M is the mass, Q is the electric charge and Λ is the cosmological constant. **(10 marks)**

4. Consider propagation of gravitational waves, presented by the metric perturbation h_{ab} in a flat background spacetime η_{ab}. Assuming the strength of the gravitational wave to be small, show that Einstein's equations reduce to the following wave equation, $\Box\bar{h}_{ab} = -16\pi G T_{ab}$, where $\bar{h}_{ab} = h_{ab} - (1/2)h\eta_{ab}$ and Lorentz gauge condition, $\partial_a\bar{h}^{ab} = 0$ has been used. **(10 marks)**

5. Show that the coordinate distances between a set of particles in the TT gauge do not change as gravitational wave passes through, but the proper distances get modified. Can you explain why this is the case with proper arguments? **(10 marks)**

6. Consider a static and spherically symmetric black hole. Find out the trajectory of a photon in this background and determine the photon sphere. Can you relate this to the Schrödinger-like equation satisfied by a perturbing massless scalar field living in this background? Also is there any property of the photon sphere that can be related to the scalar quasi-normal modes associated with this black hole spacetime? **(10 marks)**